2) Next, x^n can be calculated directly from Eq. (27) for stages 2 to $(N - 1)$. Then θ^n and x_2^n are calculated from Eqs. (16) and (7), respectively.

3) Finally, θ^N is obtained from Eq. (31) for the last stage (Nth stage). If x_2^N, calculated by substituting the computed θ^N into Eq. (7), is not zero, the procedure is repeated from the first step by assuming a new value for θ^1.

4) If the x_2^N thus obtained is zero, this sequence of θ^n is the optimal control actions sought. x_1^N can be secured from Eq.(16).

A similar working scheme may be followed by assuming a value of x_1^1 instead of θ^1.

Concluding Remarks

The maximum principle approach has the same advantage as the dynamic programming method in that it allows us to optimize rockets with three or more stages and to work with fewer assumptions than the conventional calculus method. Since there is no interpolation involved in the maximum principle solution, this method yields answers more accurate than those of the dynamic programming approach. The discrete maximum principle approach also requires much less computer memory capacity than the dynamic programming method because the solutions are not imbedded as in the case of the dynamic programming method. Usually the maximum principle algorithm requires less computational effort than dynamic programming algorithm.

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Fatigue of Aluminum with Alclad or Sprayed Coatings

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Introduction

FOR improved resistance to corrosion, aluminum products of the higher strength alloys are often produced as alclad material, the surface layers of which consist of an alloy having greater resistance to corrosion than the core. Because such cladding is limited to sheet, plate wire, and tube, some use has been made of a metallized aluminum coating sprayed on the surface of more complicated shapes. Flexural fatigue strengths obtained at Alcoa Research Laboratories (Table 3.3.1(c) of Mil HDBK-5[1]) show that alclad sheet has substantially lower fatigue strength than bare sheet. Some rotating-beam fatigue tests showed that sprayed aluminum coatings also reduced the fatigue strength.

This paper shows that differences in fatigue strengths of bare and alclad plain sheet specimens are not present to the same degree in built-up construction of these materials; i.e., the fabrication stress raisers overshadow the effect of the coating on fatigue strength.

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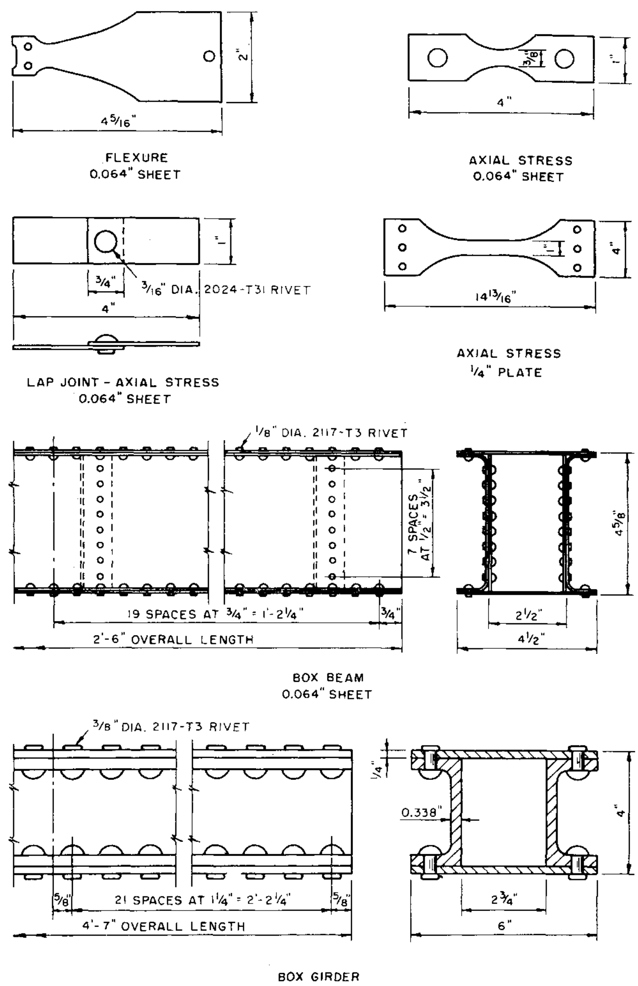

Fig. 1 Fatigue specimens.

Specimens and Test Procedures

Table 1 gives the essential data for the materials used. The details of the specimens are shown in Fig. 1. The material to be spray coated was prepared by blasting it with angular steel grit. Then a 99.5% aluminum coating was applied by an atomizing spray unit. The plate and channels for the box girders were spray coated before the riveting was accomplished.

The 0.064-in.-thick axial stress specimens were tested in 5-kip capacity Krouse direct-stress fatigue testing machines at stress ratios (R = minimum stress/maximum stress) of -1.0, -0.5, 0, 0.5, 0.75, and 0.90. The zero-stress ratio, axial-stress tests of the 0.25-in. plate were conducted in a similar 15-kip capacity machine. The sheet flexure tests were performed in Sontag SF-2 constant load flexural fatigue machines at a -1.0 stress ratio. The -1.0 stress ratio, axial-load fatigue tests of riveted lap joints were performed using special adaptors in ARL rotating-beam fatigue machines or in a 5-kip Krouse machine. The tests of lap joints and box beams were described in Refs. 2–4. The box girders and box beams were subjected to flexural tests in Templin structural fatigue machines. Most of the box beams were loaded through a 4-in.-long block located centrally on a 25-in. span. The loads were applied to the girders through fixtures attached to the webs of the channels at points 4-in. from midspan of a 40-in. span. For the box girders, R was 0, and for the box beams, R was -1.0.

The loads applied by the various fatigue machines were maintained by periodic checks. The tests were considered complete when the automatic cutoff switch on the machine stopped the test and a visible crack was observed.

Table 1 Tensile properties of aluminum alloy sheet and plate

Alloy	Types of specimens[a]	Nom. thick., in.	Cladding or coating each face, thick., %	Strengths (gross area)		Elong., 2-in., %
				Tensile, ksi	Yield, ksi	
2024-T3	*A,F,L*	0.064	None	71.6	53.1	19.5
	B	0.064	None	73.8	55.5	15.0
Alclad 2024-T3	*A,F,L*	0.064	2.7	69.1	51.0	20.6
	B	0.064	5.5	67.5	51.2	14.5
7075-T6	*A,F*	0.064	None	80.9	74.4	10.7
	L	0.064	None	80.2	72.5	10.5
Alclad 7075-T6	*A,F*	0.064	4.3	75.3	67.9	10.8
	L	0.064	2.8	80.4	71.2	14.0
2014-T4	*A,G*	0.25	None	63.9	45.2	22.0
2014-T4 Sprayed W/99% Al	*A,G*	0.25	5.5	59.1	41.2	19.2

[a] Fatigue specimens: A = axial stress, B = box beam, F = flexure, G = box girder, L = lap joint.

Results

The results of the axial-stress fatigue tests of the 0.064-in.-thick 2024-T3 and 7075-T6 bare and alclad sheet are presented in Fig. 2. Beyond 10^5 cycles, the bare specimens of either material have significantly higher fatigue strengths for all stress ratios. The advantage of the bare material generally increases as the stress decreases.

The -1.0 stress ratio results obtained for the several types of bare and alclad specimens of alloys 2024-T3 and 7075-T6 are compared in Fig. 3. The sheet flexure curves are generally higher than the axial stress curves for the same lots of material; nevertheless, the latter curves showed a larger advantage for the bare material over the alclad. The results for the riveted joints and box beams indicate that the large differences in the fatigue strength of bare and alclad material are not likely to show up in riveted construction. This result is in agreement with those found by other investigators. Forrest[5] reported that there was no difference between the fatigue strengths of riveted lap joints in bare and alclad sheet of alloy H15 (similar to 2014-T6) and that the same was true for spot welds in another aluminum-copper alloy. Dinsdale and Newman[6] showed that, whereas cladding reduced the fatigue strength of bare HS15 sheet by 50%, the reduction was only 19% for transverse butt welds in the same sheet. Thus, the advantage in fatigue strength for bare materials may be nullified by the presence of the stress concentrations inherent in actual construction.

The results of the zero stress ratio, axial-stress fatigue tests of the bare- and the spray-coated 2014-T4 plate specimens, and box girders are shown in Fig. 4. The long-life

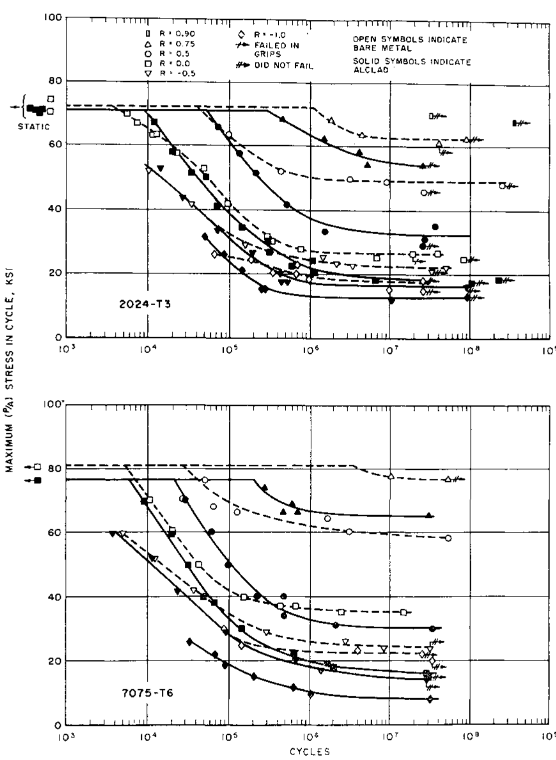

Fig. 2 Results of axial stress fatigue tests of bare and alclad sheet.

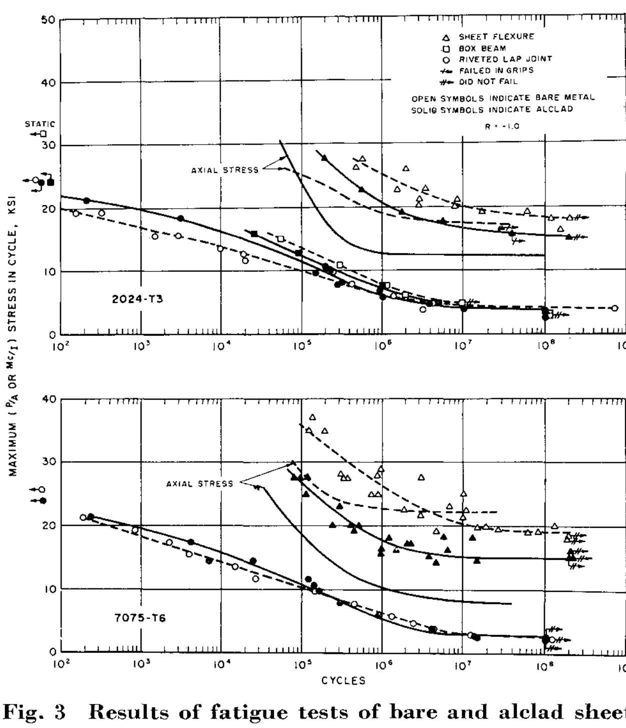

Fig. 3 Results of fatigue tests of bare and alclad sheet.

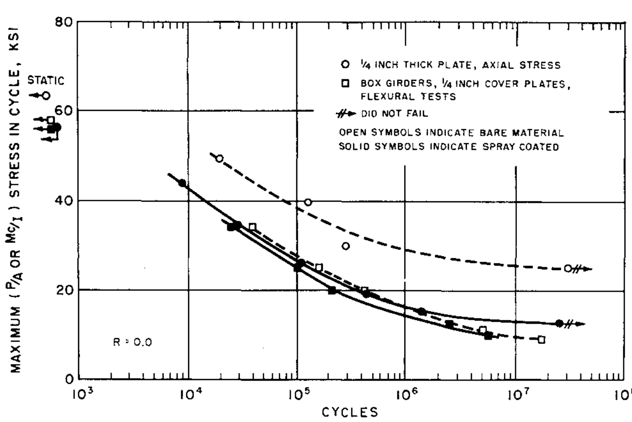

Fig. 4 Results of fatigue tests of bare and sprayed 2014-T4 plate and box girders.

fatigue strength of the sprayed plate specimens is only about half that of the bare ones. The static tests showed that the coating did not add to the strength of the base material. As illustrated in Fig. 5, metallographic examination of a few specimens revealed that fatigue cracks sometimes formed in the base metal without penetrating into the sprayed coating. Therefore, it would seem that the coating was not detrimental per se, but that the reduction in fatigue strength was caused by the notches produced by surface blasting in preparation for the coating. The spray-coated box girders had only slightly shorter lives than those of the comparably stressed bare girders, and there is little difference between the curves for the girders and the curve for the sprayed axial-stress specimens. Generally, the girder failures were initiated at rivet holes in the cover plates subjected to tensile stress. However, two sprayed girder failures were confined to the center section of a cover plate and did not involve a rivet hole. Consequently, it appears that the notches resulting from the surface blasting of the material before spraying were almost as severe as riveted construction.

Conclusions

Alclad specimens were found to have lower fatigue strengths than bare specimens in both sheet flexure and axial-stress fatigue tests. The reduction of the fatigue strength due to the cladding was as much as 50% for 7075-T6 and 33% for 2024-T3. The use of a sprayed coating on alloy 2014-T4 decreased the fatigue strength of axial-stress specimens about as much as did the cladding of the 7075-T6 material. However, the stress raisers present in riveted construction overshadowed the effect of the coatings, so that there was little difference in the fatigue strengths of beams and joints fabricated of bare and clad materials.

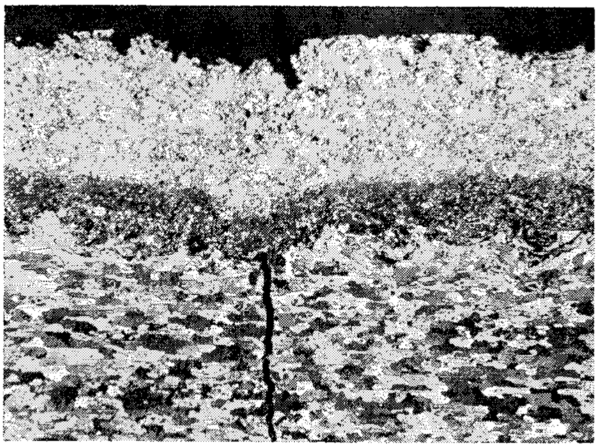

Fig. 5 Fatigue crack in base metal of spray-coated plate.

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Manned Space Station Orbiting in the Van Allen Belt

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THE basic requirement for the design of the hull structure of a space vehicle is that it provide the requisite structural integrity for the launch phase and the orbital trajectory, strength of materials being primary. For a manned space vehicle three further design considerations are vital, primarily for safety of the crew: 1) micrometeoroid protection, 2) radiation protection, and 3) heat rejection. Engineering thought regarding micrometeoroid protection has been dominated for years by the concept of the meteoroid bumper introduced by Whipple.[1] The bumper is simply a thin secondary layer of metal on the space side of the vehicle, placed at a distance from the major surface of a few thicknesses of that surface. The function is to reduce the number of punctures by exploding the meteoritic particles far enough away from the structural surface that only fragments and vapor strike it. However, for a manned space station orbiting in the Van Allen belt, the hull design must be adequate to protect the crew from the effects of the electrons in the belt. The purpose of this note is to show that, in the light of recent results on the micrometeoroid risk, this danger can be dissipated by centering the design criteria on radiation protection and external heat transfer; in particular, a separate meteoroid bumper can be dispensed with.

In a recent paper, Whipple[2] revised his figures for micrometeoroid risk, as a result of the artificial meteor experiment of Trailblazer I. This revision brings the figures inferred from photographic and visual counts of meteors more into agreement with results from satellites and space probes. The paper entirely changes the conception of the problem of the micrometeoroid danger to a space vehicle. Briefly, the main conclusion is that Whipple's former figures[1, 3] for the risk of penetration were too large by a factor of about 3000.

The impact of this revision is significant when radiation protection of humans is a design requirement.[4] In this case, a thickness of, for example, aluminum of the order of 1 cm is required to protect a human in a manned space vehicle from the electrons in the Van Allen belt.[5] This figure has been determined for an orbit at an altitude greater than about 200 km and less than a few earth radii, for a manned mission lasting several months. From the latest paper of Whipple, one finds a mean time τ for a micrometeoroid to penetrate an area of 1 m^2 of aluminum of thickness 1 cm, as tabulated in the first and second lines of Table 1, corresponding to most realistic and pessimistic of his estimates, respectively. It is obvious that, on the revised figures, astronomical times are required for one perforation in a square meter. A reasonable estimate of the total surface area of a manned orbital space station is 150 m^2, on the basis of present and projected capabilities of booster rockets. Corresponding values of the mean times for one penetration anywhere in the station are shown in the third and fourth lines of Table 1, for Whipple's most realistic and pessimistic estimates, respectively. It is noted that both times are long compared to any reasonable presumptive orbiting time of the station for human occupancy.

For a mean time τ of perforation for a particular area, the probability p_n of n penetrations in time t is given by the Poisson distribution

$$p_n = e^{-t/\tau}(t/\tau)^n/n! \tag{1}$$

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